INFLUENCE OF CONVECTION AND THERMAL RADIATION ON THE COOLING OF A VERTICALLY INCIDENT JET OF MELT

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The singularities of solidification of an axisymmetric jet of mineral melt moving in a gas medium are numerically investigated.

Let us examine the jet flow of a mineral melt in a gravity field (Fig. 1). Of practical interest are computations of the flow and cooling parameters of the accelerating jet. Optical pyrometer measurements showed that for an initial melt temperature on the order of 1500°K the surface of an incident jet of 2-3 cm diameter is cooled 50-60°K at distances of around 1 m independently of the discharge. The viscosity, density, coefficient of melt heat conductivity, and coefficient of interphasal tension here vary negligibly [1] so that the model of a viscous fluid flow with an interphasal interface can be used here [2]. In this paper we neglect the influence of compressibility and radiation on the flow of the external medium. The solution of the problem of the congealing of slowly flowing thin jets of melted glass with an initial temperature on the order of 1600°K and 1-mm diameter [3] is known from the literature. This is the other limit case of the flows examined in the present paper.

We shall examine the combined flow of the melt and the medium in a cylindrical O, r, z, φ coordinate system coupled to the jet axis (Fig. 1). The origin is at the center of the exit hole. The flow under consideration is symmetric in the azimuthal direction and the velocity in this direction is zero. We neglect mass transfer between phases. The flow characteristics to be determined are the velocity, pressure, and temperature distributions in the jet and in the environment, as well as the interphasal surface. It is shown in the paper that the influence of the medium on the jet heat elimination because of convection is slight compared to the thermal radiation of the jet surface.

1. Formulation of the Problem

The combined flow of a nonisothermal, immiscible fluid in a jet and its environment is described by the Navier—Stokes and energy equations [2]:

$$\frac{d\mathbf{u}_j}{dt} = -\nabla p_j + \frac{\nu_j}{\nu_1 \mathrm{Re}} \Delta \mathbf{u}_j + \frac{2-j}{\mathrm{Fr}} \mathbf{k}_x \tag{1}$$

$$\nabla \cdot \mathbf{u}_j = 0, \quad \frac{d\theta_j}{dt} = \frac{\varkappa_j}{\varkappa_1 \mathrm{Pe}} \Delta \theta_j \ (j=1,\ 2). \tag{2}$$

The term corresponding to Rayleigh dissipation is omitted from the energy equation since the estimates performed indicate it is small compared with the remaining components. Consequently, the hydrodynamic problem can be solved independently of the thermal problem. The equation of the unknown jet surface has the form

$$\frac{dh}{dt} = v, \quad y = h(x). \tag{3}$$

The boundary conditions are: axial symmetry of the flow, continuity of the velocities and tangential stresses, a jump in the normal stresses caused by surface tension, continuity of the temperature and heat flux on the jet surface, and the condition of making the transition into the uniform unperturbed flow in the environment far from the jet surface is satisfied. These conditions can be represented in the form

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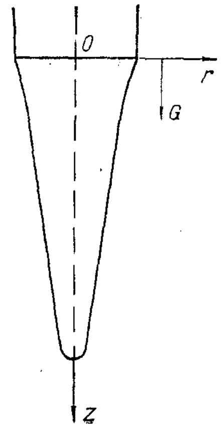

Fig. 1. Schematic diagram of the jet of melt.

$$v_1 = u_{1y} = \theta_{1y} = 0,\ y = 0, \tag{4}$$

$$[u]_2^1 = 0,\ [v]_2^1 = 0, \tag{5}$$

$$\left[\frac{\mu}{\mu_1}\left\{u_y + v_x + 2b\left(2v_y + \frac{v}{y}\right)\frac{1}{1-b^2}\right\}\right]_2^1 = 0, \tag{6}$$

$$\left[-\frac{\rho}{\rho_1}p + 2\frac{\nu}{\nu_1}\left\{(1+b^2)v_y + b^2\frac{v}{y}\right\}\frac{1}{(1-b^2)\mathrm{Re}}\right]_2^1 = -\frac{2}{\mathrm{We}\,R_S}, \tag{7}$$

$$[\theta]_2^1 = 0, \tag{8}$$

$$\left[\frac{\lambda}{\lambda_1}\frac{\theta_y - b\theta_x}{\sqrt{1+b^2}} + \varepsilon\,\mathrm{Bo}\left(\theta + \frac{T_E^0}{\Delta T}\right)^4\right]_2^1 = 0, \tag{9}$$

$$u_2 \to U_E(x),\ \theta_2 \to \theta_E(x),\ y \to \infty\ (|b| < 1). \tag{10}$$

The square brackets here denote the jump in the appropriate quantity on the interphasal surface. Conditions (5)-(9) are satisfied for $y = h(x)$. The boundary-value problem (1)-(10) should be supplemented by conditions at $x = 0$ and $x \to \infty$. Values of the dimensionless parameters of the problem as well as of the independent and dependent variables are determined from the formulas:

$$\frac{1}{\mathrm{Fr}} = \frac{(1-\rho_0)GR}{U^2},\ \mathrm{Re} = \frac{\rho_1 RU}{\mu_1}, \tag{11}$$

$$\mathrm{Pe} = \frac{\rho_1 C_{V1} RU}{\lambda_1},\ \mathrm{Bo} = \frac{\sigma R(\Delta T)^3}{\lambda_1},\ \mathrm{We} = \frac{\gamma}{\rho_1 U^2 R},$$

$$(x,\ y) = (z,\ r)R^{-1},\ (u,\ U_E,\ v) = (u_*,\ U_{*E},\ v_*)U^{-1},$$

$$\varkappa_j = \frac{\lambda_j}{\rho_j C_{Vj}},\ p_j = \frac{p_{*j}}{\rho_j U^2},\ \rho_0 = \frac{\rho_2}{\rho_1}\ (j = 1,\ 2),$$

$$\theta = \frac{T - T_E^0}{\Delta T},\ \Delta T = T^0 - T_E(0),\ T_E^0 = T_E(0), \tag{12}$$

$$\frac{2}{R_S} = \left\{\frac{1}{h} - \frac{\dot{b}}{1+b^2}\right\}\frac{1}{\sqrt{1+b^2}},\quad b = \dot{h}(x) = \frac{dh}{dx}.$$

The asterisk denotes dimensional quantities. In relationships (11)-(12), U is the maximal escape velocity of the melt through a nozzle of radius R; G, acceleration of gravity; T^0, maximal jet temperature at $z = 0$; $T_E(z)$, temperature of the environment; $u_* = \{u_*,\ v_*\}$, axial and radial velocity components; p_*, pressure; R_S, effective radius of curvature of the jet surface; $\sigma = 5.67 \cdot 10^{-8}$ W/m²·deg⁴, emissivity of an absolute blackbody; ε_j $(j = 1,\ 2)$, dimensionless coefficient of grayness of the jet and medium surfaces, respectively; Δ, Laplace operator; ∇, gradient operator; and d/dt, substantive derivative.

The boundary condition

$$\frac{\theta_y - b\theta_x}{\sqrt{1+b^2}} + \mathrm{Bi}_x\,\theta = 0,\; y = h(x) \tag{13}$$

is used in place of (9) in engineering computations to describe the heat elimination of a jet surface moving in a gas ($b \equiv 0$ [1] in the case of a cylindrical jet). Here

$$\mathrm{Bi}_x = \frac{\alpha_x R}{\lambda_1} = \frac{\lambda_2}{\lambda_1}\mathrm{Nu}_x = \lambda_0 \mathrm{Nu}_x,$$
$$\mathrm{Nu}_x = 0.0205\,\mathrm{Pr}^{0.4}\,\mathrm{Re}_x,\; \mathrm{Re}_x = \frac{2\rho_2 h_*(r)\,u_*(z,\,h_*)}{\mu_2}, \tag{14}$$

which describe the criterial dependence of the heat elimination coefficient of the heated surface of a cylindrical tube of radius $h_*(z)$, equal to the local value of the jet radius, and the gas flow whose velocity equals the local value $u_*(z, h_*)$, and the velocity of the jet surface relative to the fixed medium [4]. In (14), α_x is the coefficient of heat elimination of the interphasal surface, Pr is the Prandtl number. It is shown below that the dynamic influence of the medium on the interphasal surface can be neglected in a number of cases, and the boundary conditions

$$u_{1y} + v_{1x} + \frac{2b}{1-b^2}\left(2v_{1y} + \frac{v_1}{y}\right) = 0, \tag{15}$$

$$p_1 - \rho_0 p_2 = \frac{2}{\mathrm{We}\,R_S} + \frac{2}{(1-b^2)\,\mathrm{Re}}\left\{(1+b^2)\,v_{1y} + \frac{b^2 v_1}{y}\right\} \tag{16}$$

can be considered in place of (5)-(7).

The problem (1)-(10) has a nontrivial solution in the case of a free cylindrical jet of constant radius [5]:

$$h \equiv 1,\; u_1(x,\,y) \equiv u_1(y),\; v_1 \equiv 0,\; \theta_1(x,\,y) \equiv \theta_1(y).$$

Under real conditions, the fluid in the jet is subjected to the action of mass forces, interphasal tension, and the external medium, which implies acceleration and redistribution of the velocity across the jet, and therefore, a deviation of its shape from the cylindrical.

2. Flow Investigation Method

When the parameters Re, Pe $\gg 1$, and the radial velocity component in the initial section is small, the change in the solution across the jet is much greater than in the longitudinal direction, i.e.,

$$\frac{\partial}{\partial x} << \frac{\partial}{\partial y}.$$

To the accuracy of higher-order infinitesimals, problem (1)-(10) can be represented in the form

$$u_1 u_{1x} + v_1 u_{1y} = -p_{1x} + \frac{(y u_{1y})_y}{y\,\mathrm{Re}} + \frac{1}{\mathrm{Fr}},$$
$$0 = -p_{1y},\; y u_{1x} + (y v_1)_y = 0, \tag{17}$$

$$u_1\theta_{1x} + v_1\theta_{1y} = \frac{(y\theta_{1y})_y}{y\,\mathrm{Pe}}, \tag{18}$$

$$u_2 u_{2S} + v_2 u_{2n} = -p_{2S} + \frac{\nu_0 (h_1 u_{2n})_n}{h_1 \mathrm{Re}}, \tag{19}$$

$$0 = -p_{2n},\ (h_1 u_2)_S + (h_1 v_2)_n = 0, \tag{20}$$

$$u_2 \theta_{2S} + v_2 \theta_{2n} = \frac{\varkappa_0 (h_1 \theta_{2n})_n}{h_1 \mathrm{Pe}}, \tag{21}$$

$$u_1 \dot{h} = v_1, \quad y = h(x),$$

$$u_{1y} = 0,\ y = 0,\ u_1 = u_2,\ u_{1y} = \mu_0 u_{2n},\ p_1 = \rho_0 p_2 + \frac{1}{h\,\mathrm{We}},\ y = h(x), \tag{22}$$

$$\theta_{1y} = 0,\ y = 0,\ \theta_1 = \theta_2,\ \theta_{1y} = \lambda_0 \theta_{2n} - \varepsilon_1 \mathrm{Bo}\left(\theta + \frac{T_E^0}{\Delta T}\right)^4,\ y = h(x), \tag{23}$$

$$u_2 \to U_E(x),\ \theta_2 \to \theta_E(x),\ y \to \infty, \tag{24}$$

$$\nu_0 = \frac{\nu_2}{\nu_1},\ \mu_0 = \frac{\mu_2}{\mu_1},\ \varkappa_0 = \frac{\varkappa_2}{\varkappa_1},$$

$$u_1(0, y) = U_1(y),\ u_2(0, n) = U_2(n),\ h(0) = 1, \tag{25}$$

$$\theta_1(0, y) = \Xi_1(y),\ \theta_2(0, n) = \Xi_2(n). \tag{26}$$

The flow of the external medium is here considered in an internal orthonormal coordinate system S, n, φ coupled to the interface [6], and $h_1(S, n)$ is the Lame parameter,

(27)

To infinitesimal accuracy $O(\dot{h}_S^2)$ it can be assumed that cos β = 1, S = x. For T_E 310°K the thermal radiation of the medium in (9) is negligible, and the corresponding term in (23) is omitted.

We use the method of equal discharge surfaces [6, 7] to compute the combined flow of a heated jet and the environment. We will solve the energy equation simultaneously with computing the flow field. We reduce (19) to a system of ordinary differential equations to determine the values of the solution on the stream surfaces $y^{(m)}(x)$:

$$u_1^{(m)} \dot{\theta}_1^{(m)} = \frac{1}{\mathrm{Pe}}\left(\theta_{1yy} + \frac{\theta_{1y}}{y}\right),\ \theta_1^{(m)}(0) = \Xi_1^{(m)} \quad (m = 1, 2, \ldots, M). \tag{28}$$

Here the dot denotes differentiation with respect to the axial coordinate. As in the flow computation, we represent the reduced temperature in the jet in the form of a sum of pairs of components. The former corresponds to the homogeneous condition on the axis and the inhomogeneous condition on the jet surface. We approximate it by using the very same system of functions $V_\mu(x, y)$ that is used to solve the hydrodynamic problem, and we take a quadratic as the latter component:

$$V_1 \equiv 1,\ V_\mu(x, y) = \eta^{2\mu-2}\left(1 - \frac{\mu - 1}{\mu}\eta^2\right),\quad \eta = \frac{y}{h(x)},$$
$$\theta_1(x, y) = \frac{d_0(x)\, y^2}{h(x)} + \sum_{\mu=1}^{M} d_\mu(x) V_\mu(\eta). \tag{29}$$

The coefficients $d_\mu(x)$ are selected from the condition that (29) agrees with the exact solution $\theta_1{}^{(m)}(x)$ on the stream surfaces $y^{(m)}(x)$ computed by means of (2.10) and (2.11) in [6]

TABLE 1. Values of the Dimensionless Temperature on Jet Surface

x	ε_1	$Q_h\cdot10^{-3}$ kg/h	$\theta_S=\theta(x, h)$	
			$\rho_1=2.5\cdot10^3$ kg/m³	$\rho_1=3\cdot10^3$ kg/m³
80	1	2	0,92432	0,92980
		1	0,92088	0,92659
80	0,9	2	0,93065	0,93542
		1	0,92734	0,93266
80	0,8	2	0,93715	0,94182
		1	0,93401	0,93892
80	0,7	2	0,94387	0,94809
		1	0,94093	0,94538
80	0,6	2	0,95080	0,95455
80	0,0	2	0,99775	—
		1	0,99747	—

T° = 1593 K; T_E = 293 K; ν_0 = 0.04183; ρ_0 = $5.172\cdot10^{-4}$; λ_0 = $1.784\cdot10^{-2}$; U_E = 0; R = 0.015 m (U = 0.314 m/sec; Fr = 1.4873; Re = 13.10; Bo = 0.8997; Pe_1 = $9.8464\cdot10^3$; Pe_2 = $2.3746\cdot10^2$). Q_h — hourly melt discharge.

and by (28) from this paper. Determination of $d_\mu(x)$ reduces to solving a system of linear algebraic equations in successive sections. The matrix of the system mentioned is exactly the same as in the velocity computation, because of the selection of the functions $V_\mu(\eta)$. Consequently, it is inverted just once in each spacing in x and the computation of the flow and the solidification of the melt jet is performed without substantial enlargement of the volume of calculations.

3. Flow of a Jet with a Uniform Velocity Profile

Results of computing jet flows in a gas permitted us to obtain a formula to estimate the size of the domain for a uniform velocity profile build-up in a jet $L_r \approx 0.1\cdot Re$ [6, 7]. The dynamic influence of the medium on the fluid flow in the melt jet can be neglected since the value is $\mu_0 \approx 2\cdot10^{-5}$ and the fluid velocity is 4-5 m/sec at a range of 1-1.5 m from the melt supply site. Hence, for $x \geqslant L_r$ the velocity distribution in the jet should remain almost uniform.

Let us seek the solution of problem (17), (21), (22), (25) for $x > L_r$ in the form $u_1 = u(x)$. Equations (17), (21) and the boundary conditions (25) permit a problem with initial conditions to be obtained

$$u\dot{u}=\frac{1}{\mathrm{Fr}},\quad v=-0.5y\dot{u},\quad u\dot{h}=-0.5h\dot{u}. \tag{30}$$

The dot in (30) denotes differentiation with respect to x. We represent the solution of (30) in the form

$$u(x)=\sqrt{1+\frac{2x}{\mathrm{Fr}}},\quad v=-\frac{y}{2\,\mathrm{Fr}\sqrt{1+\dfrac{2x}{\mathrm{Fr}}}},$$
$$h=\frac{1}{\sqrt{u}},\quad u(0)=1,\quad h(0)=1. \tag{31}$$

We modify the thermal problem (18), (20), (23), (24) with (31) taken into account. We execute the change of variables X = x, Y = y/h(x), and transform the derivatives according to the formulas

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial X}-Y\frac{\dot{h}}{h}\frac{\partial}{\partial Y}=\frac{\partial}{\partial X}+\frac{Y}{2}\frac{\dot{u}}{u}\frac{\partial}{\partial Y},\quad \frac{\partial}{\partial y}=\frac{1}{h}\frac{\partial}{\partial Y}. \tag{32}$$

We substitute (32) into (18) and the boundary conditions (23). After manipulation, we obtain the boundary-value problem

$$\theta_{1X}=\frac{1}{\mathrm{Pe}}\left(\theta_{1YY}+\frac{\theta_{1Y}}{Y}\right),\ \theta_{1Y}=0,\ Y=0, \tag{33}$$

$$\theta_{1Y}=h(X)\left\{\lambda_0\theta_{2n}-\varepsilon_1\,\mathrm{Bo}\left(\theta_2+\frac{T_E^0}{\Delta T}\right)^4\right\},\ Y=1. \tag{34}$$

In the case when the heat-elimination condition on the jet surface is taken in form (13), (14), the boundary condition (34) with radiation taken into account is converted to the following:

$$\theta_{1Y}=-h(X)\left\{\mathrm{Bi}_x\theta_1+\varepsilon_1\,\mathrm{Bo}\left(\theta_1+\frac{T_E^0}{\Delta T}\right)^4\right\},\ Y=1. \tag{35}$$

The thermal problem (33), (35) was numerically solved by the method of lines [8], with (31) taken into account. For fixed R and U, the computations were performed for different values of the grayness index ε_1 = 0; 0.4; 0.6; 0.7; 0.8; 0.9; 1. The results of the solution are presented in Table 1 for $\Xi_1(y)\equiv 1$. For ε_1 = 0 cooling of the jet surface is slight. As ε_1 increases, the drop in temperature increases. The radial temperature distribution in the jet differs from the uniform distribution only near the interphasal surface. A relative change in the temperature at x = 80 is 0.2-0.3% for a twofold change in the melt supply. Cooling of the jet surface in the case $\rho_1 = 3\cdot10^3$ kg/m³ is less than for $\rho_1 = 2.5\cdot10^3$ kg/m³. The change in the jet surface temperature is

$$T^0-T(z,\ h_*)=\{1-\theta_1(x,\ h)\}\,\Delta T.$$

The temperature drop will be greater, the greater the magnitude of the thermal head ΔT.

Presented below, for comparison, is the solution of the adjoint thermal problem (31), (33), (34), (19), (20), (23)-(26) in the case of a jet with uniform profile flowing in a fixed medium.

4. Description of the Flow and Heat Transfer in the Medium

To describe the flow of the medium in the transition domain near the interphasal surface, we use the method of integral relations. The formulas to compute the thickness of the dynamic layer $\delta_u(S)$ for a parabolic longitudinal velocity distribution are derived in [6]. Approximating the temperature profile in the thermal layer by a quadratic trinomial and integrating (19) with respect to n between the limits $0\leqslant n\leqslant\delta_\theta(S)$, we obtain an equation to determine the thickness of the thermal layer

$$\dot\delta_\theta=\frac{1}{A_p}\left\{\frac{2h\varkappa_0 g(S)}{\delta_\theta\,\mathrm{Pe}}-L_p\right\},\ \ \delta_\theta(0)=\delta_\theta^0. \tag{36}$$

The following notation is used in the right side of (36):

$$A_p=\begin{cases}\dfrac{1}{6}U_E\,g(2h+\delta_\theta)-A-\omega(C+\delta_\theta E), & \omega\leqslant 1,\\[2mm] \dfrac{1}{30}\{5U_E\,g(2h+\delta_\theta)-2(\delta_\theta+3h)fg\}, & \omega>1,\end{cases}$$

$$L_p=\begin{cases}\dfrac{U_E\,\delta_\theta}{12}\{4(gh)^{\cdot}+\dot g\delta_\theta\}-\delta_\theta(B+D\delta_\theta)+\dot\delta_u\omega^2(C+\delta_\theta E), & \omega\leqslant 1,\\[2mm] \dfrac{U_E\,\delta_\theta}{12}\{4(gh)^{\cdot}+\dot g\,\delta_\theta\}-\dfrac{\delta_\theta}{30}\{(fg)^{\cdot}(6h+\delta_\theta)+6f\dot g h\}, & \omega>1,\end{cases}$$

$$A=G_1+2\delta_\theta G_2,\ B=z_1(fgh)^{\cdot},\ C=fgh\,\frac{2\omega-5}{30},\ D=z_2(fgh)^{\cdot},$$

$$E=fgh\frac{\omega-2}{30},\ G_1=z_1fgh,\ G_2=z_2fg,\ \omega=\delta_\theta/\delta_u,$$

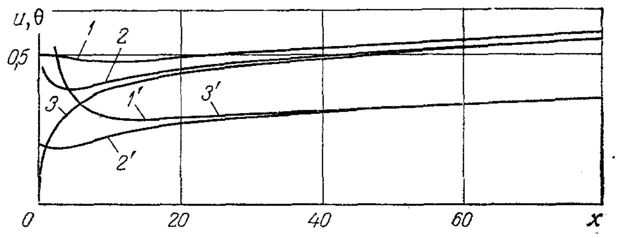

Fig. 2. Thermal and dynamic layer thicknesses.

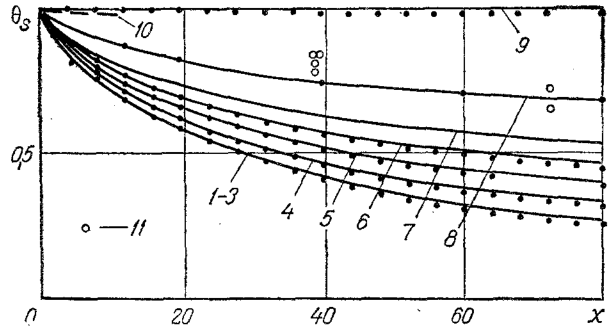

Fig. 3. Dependence of the reduced temperature on the jet surface for different grayness coefficients: 1-3) $\varepsilon_1 = 1$; 4) 0.9; 5) 0.8; 6) 0.7; 7) 0.6; 8) 0.4; 9) 0; 10) computation taking account of the velocity profile change; 11) experimental data under conditions corresponding to the tabulated results. $Q_h = 2 \cdot 10^3$ kg/h.

$$z_1 = \frac{10 + (\omega - 5)\,\omega}{30}, \quad z_2 = \frac{5 + (\omega - 4)\,\omega}{60},$$

$$f(S) = U_E(S) - U_1(S, h); \; g(S) = \theta_E(S) - \theta_1(S, h).$$

Here $\dot{U}_E = 0$, $\dot{\theta}_E = 0$.

For completeness, we present the problem to determine $\delta_u(S)$:

$$(2\delta_u L_{20} + L_{10})\,\dot{\delta}_u = \frac{2\nu_0 h}{\delta_u \,\mathrm{Re}} - \frac{\delta_u^2 \dot{L}_2 + \delta_u \dot{L}_1 + U_E I_0}{f},$$

$$L_j = f(S) L_{j0} \; (j = 1,\ 2),\ L_{10} = \left(\frac{U_E}{3} - \frac{f}{5}\right) h,\ L_{20} = \frac{U_E}{12} - \frac{f}{30}, \tag{37}$$

$$I_0 = \left(\frac{h}{3} + \frac{\delta_u}{12}\right) \delta_u f,\ \delta_u(0) = \delta_u^0,$$

$$u_2(S, n) = U_E(S) - f(S)\left\{1 - \frac{n}{\delta_u(S)}\right\}^2,\ \theta_2(S, n) = \theta_E(S) - g(S)\left\{1 - \frac{n}{\delta_\theta(S)}\right\}^2. \tag{38}$$

5. Computation Results

The problem (31), (33), (34), (36), (37) with initial data was numerically solved by the Runge–Kutta method. The results of the solution are represented in Figs. 2-4. Values of the thickness of the thermal and dynamic layers are shown in Fig. 2 for different values of $\delta_u{}^\circ$, $\delta_\theta{}^\circ$. Curves 1-3 are $\delta_\theta(S)$, while 1'-3' are $\delta_u(S)$, respectively, for the following initial values of the layer thickness: 1') $\delta_u{}^\circ = 2$, $\delta_\theta{}^\circ = 0.5$; 2') $\delta_u{}^\circ = 0.2$, $\delta_\theta{}^\circ = 0.5$; 3) $\delta_u{}^\circ = 2$, $\delta_\theta{}^\circ = 0.05$.

Starting with a certain x, the thermal layer thickness is greater than the dynamic layer thickness independently of their initial values. For sufficiently large x the sizes of the appropriate layers are close in magnitude. The dependence of the reduced temperature on

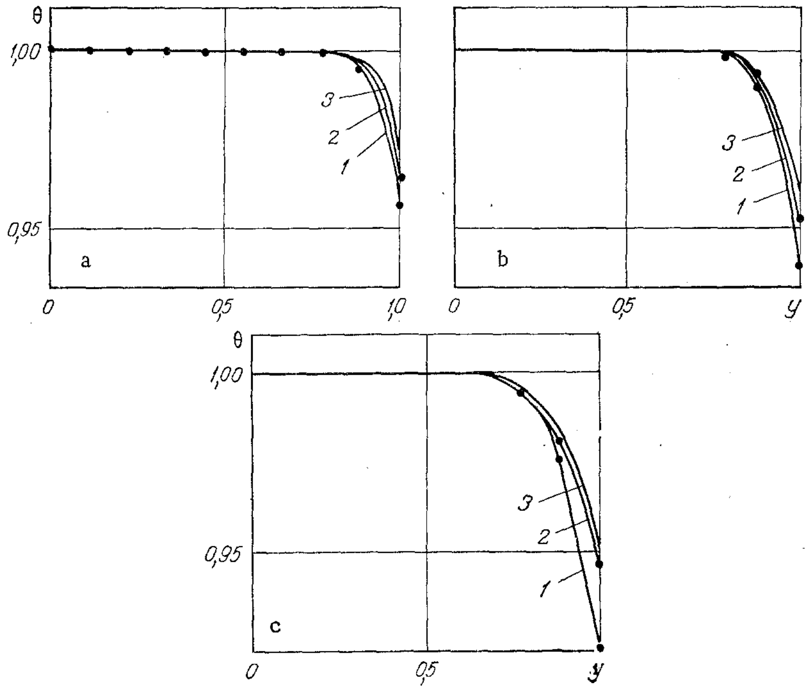

Fig. 4. Temperature profiles across the jet at different sections: a) x = 20; b) 40; c) 80; 1) ε_1 = 1.0; 2) 0.8; 3) 0.6.

the jet surface on x is shown in Fig. 3 for different values of the grayness coefficient ε_1. As is seen from the figure, the results of the solution obtained by all three methods differ insignificantly. The temperature profiles across the jet are represented in Fig. 4 for the sections x = 20, x = 40, x = 80 ($\Xi_1(y)\equiv 1$ in the initial section).

Computations were performed of congealing of the melt jet with nonuniform initial velocity and temperature distributions without taking account of the thermal radiation. The initial profiles used were

$$F_0(y)=\varepsilon+(1-2\varepsilon)(1-y^2),\ \varepsilon=0,\ 1,\ 0\leqslant y\leqslant 1, \tag{39}$$

$$F_0(y)=1-1.8y^2\left(1-\frac{y^2}{2}\right). \tag{40}$$

The slight influence of the selection of $\delta_u{}^{\circ}$, $\delta_\theta{}^{\circ}$ on the final velocity and temperature distribution in the jet and in air was shown first. Values of the solutions for x = 4 were compared in the following cases

$$\delta_\theta^0=0.05,\ \delta_u^0=0.01;\ \delta_\theta^0=0{,}05,\ \delta_u^0=0.001 \text{ and}$$

$$\delta_\theta^0=0{,}01,\ \delta_u^0=0{,}01.$$

The velocity profile in the form (39) was selected as initial profile. The initial temperature profile was taken uniform $\Xi_1(y)\equiv 1$.

Results of computations of a jet with uniform initial velocity profile and initial temperature distribution (40), obtained by the method of lines [8], as well as by the method of equal discharge surfaces, turned out to be close to each other.

Computations performed by using the method of [6, 7] exhibited a slight dependence of the flow field on the initial velocity profiles with nearby mean-mass values. The initial temperature distribution has substantial influence on the nature of jet solidification. In this series of computations, the initial velocity and temperature distributions were selected both uniform and also in the form of (39) and (40).

NOTATION

$\mathbf{u}\{u, v, 0\}$, velocity vector; p, pressure; U, T°, maximal velocity and maximal jet temperature upon escaping from a nozzle of radius R; U_E, T_E, velocity and temperature of the external medium; $U_0(y)$, $\Xi_0(y)$, initial velocity and temperature distributions; θ, reduced temperature; $y = h(x)$, equation of the jet surface; δ_u, δ_θ, thicknesses of the dynamic and thermal layers in the medium; γ, coefficient of interphasal tension; μ_j, ν_j, ρ_j, λ_j, Cv_j, ε_j, dynamic and kinematic viscosity, the density, the heat conductivity coefficient and specific heat of the jet ($j = 1$) and the medium ($j = 2$), as well as the grayness coefficient of the jet and surrounding gas surfaces; σ, Stefan–Boltzmann radiation constant; Re, Pe, Pr, Fr, We, Bo, Bi_x, Nu_x, ν_0, ρ_0, λ_0, $\varkappa_0$, dimensionless parameters of the problem.

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